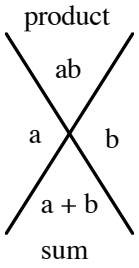
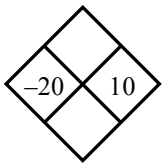
DIAMOND PROBLEMS 2.1.1

In every Diamond Problem, the product of the two side numbers (left and right) is the top number and their sum is the bottom number.

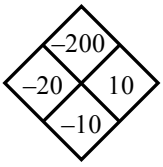
Diamond Problems are an excellent way of practicing addition, subtraction, multiplication, and division of positive and negative integers, decimals and fractions. They have the added benefit of preparing students for factoring binomials in algebra.



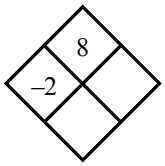
Example 1



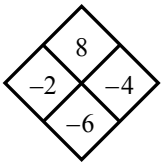
The top number is the product of –20 and 10, or –200. The bottom number is the sum of –20 and 10, or –20 + 10 = –10.



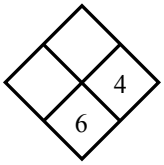
Example 2



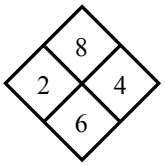
The product of the right number and –2 is 8. Thus, if you divide 8 by – 2 you get – 4, the right number.
The sum of –2 and – 4 is – 6, the bottom number.



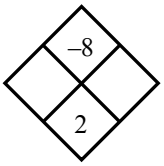
Example 3



To get the left number, subtract 4 from 6,
6 – 4 = 2. The product of 2 and 4 is 8, the top number.



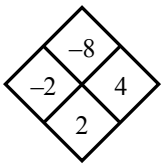
Example 4



The easiest way to find the side numbers in a situation like this one is to look at all the pairs of factors of –8. They are:

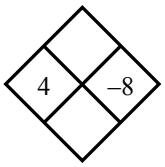
–1 and 8, –2 and 4, –4 and 2, and –8 and 1.

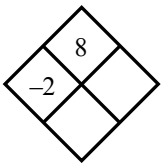
Only one of these pairs has a sum of 2: –2 and 4. Thus, the side numbers are –2 and 4.

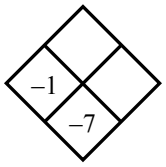


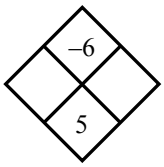
Problems

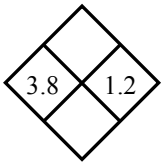
Complete each of the following Diamond Problems.

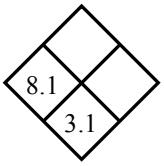
1. 

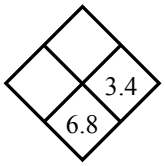
2. 

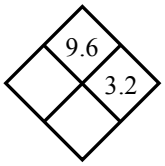
3. 

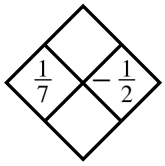
4. 

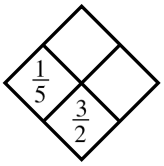
5. 

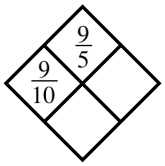
6. 

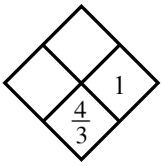
7. 

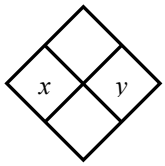
8. 

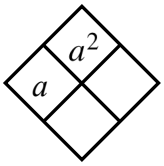
9. 

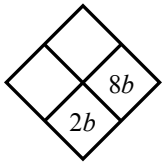
10. 

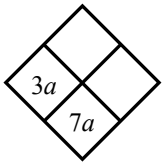
11. 

12. 

13. 

14. 

15. 

16. 

Answers

1. -32 and -4
2. -4 and -6
3. -6 and 6
4. 6 and -1
5. 4.56 and 5
6. 5 and 40.5
7. 3.4 and 11.56
8. 3 and 6.2
9. $-\frac{1}{14}$ and $-\frac{5}{14}$
10. $\frac{13}{10}$ and $\frac{13}{50}$
11. $\frac{1}{2}$ and $\frac{7}{5}$
12. $\frac{1}{3}$ and $\frac{1}{3}$
13. xy and $x + y$
14. a and $2a$
15. $-6b$ and $-48b^2$
16. $4a$ and $12a^2$

ARITHMETIC OPERATIONS WITH DECIMALS

ADDING AND SUBTRACTING DECIMALS: Write the problem in column form with the decimal points in a vertical column. Write in zeros so that all decimal parts of the number have the same number of digits. Add or subtract as with whole numbers. Place the decimal point in the answer aligned with those above.

MULTIPLYING DECIMALS: Multiply as with whole numbers. In the product, the number of decimal places is equal to the total number of decimal places in the factors (numbers you multiplied). Sometimes zeros need to be added to place the decimal point.

DIVIDING DECIMALS: When dividing a decimal by a whole number, place the decimal point in the answer space directly above the decimal point in the number being divided. Divide as with whole numbers. Sometimes it is necessary to add zeros to the number being divided to complete the division.

When dividing decimals or whole numbers by a decimal, the divisor must be multiplied by a power of ten to make it a whole number. The dividend must be multiplied by the same power of ten. Then divide following the same rules for division by a whole number.

For additional information, see the Math Notes boxes in Lessons 3.3.2 and 3.3.3 of the *Core Connections, Course 2* text.

Example 1

Add 47.37, 28.9, 14.56, and 7.8.

$$\begin{array}{r} 47.37 \\ 28.9\mathbf{0} \\ 14.56 \\ +\ 7.8\mathbf{0} \\ \hline 98.63 \end{array}$$

Example 2

Subtract 198.76 from 473.2.

$$\begin{array}{r} 473.2\mathbf{0} \\ -\ 198.76 \\ \hline 274.44 \end{array}$$

Example 3

Multiply 27.32 by 14.53.

$$\begin{array}{rl} 27.32 & \text{(2 decimal places)} \\ \times\ 14.53 & \text{(2 decimal places)} \\ \hline 8196 & \\ 13660 & \\ 10928 & \\ 2732 & \\ \hline 396.9596 & \text{(4 decimal places)} \end{array}$$

Example 4

Multiply 0.37 by 0.0004.

$$\begin{array}{rl} 0.37 & \text{(2 decimal places)} \\ \times\ 0.0004 & \text{(4 decimal places)} \\ \hline \mathbf{0.000}148 & \text{(6 decimal places)} \end{array}$$

Example 5

Divide 32.4 by 8.

$$\begin{array}{r} 4.05 \\ 8\overline{)\,32.4\mathbf{0}} \\ 32 \\ \hline 0\ 40 \\ 40 \\ \hline 0 \end{array}$$

Example 6

Divide 27.42 by 1.2. First multiply each number by 10^1 or 10.

$$1.2\overline{)27.42} \Rightarrow 12\overline{)274.2} \Rightarrow 12\overline{)274.20}$$

$$\begin{array}{r} 22.85 \\ 12\overline{)\,274.20} \\ 24 \\ \hline 34 \\ 24 \\ \hline 102 \\ 96 \\ \hline 60 \\ 60 \\ \hline 0 \end{array}$$

Problems

1. $4.7 + 7.9$
2. $3.93 + 2.82$
3. $38.72 + 6.7$
4. $58.3 + 72.84$
5. $4.73 + 692$
6. $428 + 7.392$
7. $42.1083 + 14.73$
8. $9.87 + 87.47936$
9. $9.999 + 0.001$
10. $0.0001 + 99.9999$
11. $0.0137 + 1.78$
12. $2.037 + 0.09387$
13. $15.3 + 72.894$
14. $47.9 + 68.073$
15. $289.307 + 15.938$
16. $476.384 + 27.847$
17. $15.38 + 27.4 + 9.076$
18. $48.32 + 284.3 + 4.638$
19. $278.63 + 47.0432 + 21.6$
20. $347.68 + 28.00476 + 84.3$
21. $8.73 – 4.6$
22. $9.38 – 7.5$
23. $8.312 – 6.98$
24. $7.045 – 3.76$
25. $6.304 – 3.68$
26. $8.021 – 4.37$
27. $14 – 7.431$
28. $23 – 15.37$
29. $10 – 4.652$
30. $18 – 9.043$
31. $0.832 – 0.47$
32. $0.647 – 0.39$
33. $1.34 – 0.0538$
34. $2.07 – 0.523$
35. $4.2 – 1.764$
36. $3.8 – 2.406$
37. $38.42 – 32.605$
38. $47.13 – 42.703$
39. $15.368 + 14.4 – 18.5376$
40. $87.43 – 15.687 – 28.0363$
41. $7.34 \cdot 6.4$
42. $3.71 \cdot 4.03$
43. $0.08 \cdot 4.7$
44. $0.04 \cdot 3.75$
45. $41.6 \cdot 0.302$
46. $9.4 \cdot 0.0053$
47. $3.07 \cdot 5.4$
48. $4.023 \cdot 3.02$
49. $0.004 \cdot 0.005$
50. $0.007 \cdot 0.0004$
51. $0.235 \cdot 0.43$
52. $4.32 \cdot 0.0072$
53. $0.0006 \cdot 0.00013$
54. $0.0005 \cdot 0.00026$
55. $8.38 \cdot 0.0001$
56. $47.63 \cdot 0.000001$
57. $0.078 \cdot 3.1$
58. $0.043 \cdot 4.2$
59. $350 \cdot 0.004$
60. $421 \cdot 0.00005$

Divide. Round answers to the hundredth, if necessary.

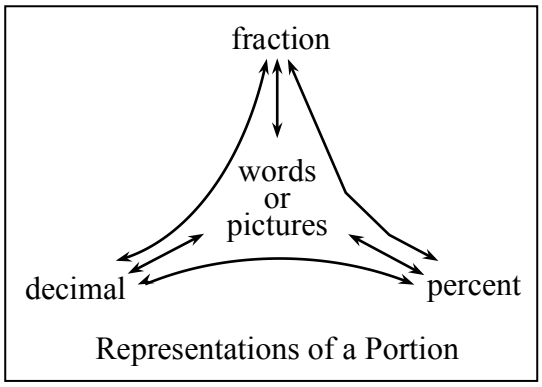
61. $14.3 \div 8$
62. $18.32 \div 5$
63. $147.3 \div 6$
64. $46.36 \div 12$
65. $100.32 \div 24$
66. $132.7 \div 28$
67. $47.3 \div 0.002$
68. $53.6 \div 0.004$
69. $500 \div 0.004$
70. $420 \div 0.05$
71. $1.32 \div 0.032$
72. $3.486 \div 0.012$
73. $46.3 \div 0.011$
74. $53.7 \div 0.023$
75. $25.46 \div 5.05$
76. $26.35 \div 2.2$
77. $6.042 \div 0.006$
78. $7.035 \div 0.005$
79. $207.3 \div 4.4$
80. $306.4 \div 3.2$

Answers

1. 12.6	2. 6.75	3. 45.42	4. 131.14	5. 696.73
6. 435.392	7. 56.8383	8. 97.34936	9. 10.000	10. 100.0000
11. 1.7937	12. 2.13087	13. 88.194	14. 115.973	15. 305.245
16. 504.231	17. 51.856	18. 337.258	19. 347.2732	20. 459.98476
21. 4.13	22. 1.88	23. 1.332	24. 3.285	25. 2.624
26. 3.651	27. 6.569	28. 7.63	29. 5.348	30. 8.957
31. 0.362	32. 0.257	33. 1.2862	34. 1.547	35. 2.436
36. 1.394	37. 5.815	38. 4.427	39. 11.2304	40. 43.7067
41. 46.976	42. 14.9513	43. 0.376	44. 0.15	45. 12.5632
46. 0.04982	47. 16.578	48. 12.14946	49. 0.000020	50. 0.0000028
51. 0.10105	52. 0.031104	53. 0.000000078	54. 0.000000130	55. 0.000838
56. 0.0004763	57. 0.2418	58. 0.1806	59. 1.4	60. 0.02105
61. 1.7875 or 1.79	62. 3.664 or 3.66	63. 24.55	64. $3.86\overline{3}$ or 3.86	65. 4.18
66. 4.74	67. 23,650	68. 13,400	69. 125,000	70. 8400
71. 41.25	72. 29.05	73. 4209.09	74. 2334.78	75. 5.04
76. 11.98	77. 1007	78. 1407	79. 47.11	80. 95.75

FRACTION-DECIMAL-PERCENT EQUIVALENTS 2.1.1 and 2.1.2

Fractions, decimals, and percents are different ways to represent the same portion or number.



Representations of a Portion

For additional information, see the Math Notes box in Lesson 2.1.2 of the *Core Connections, Course 2* text. For additional examples and practice, see the *Core Connections, Course 2* Checkpoint 2 materials.

Examples

Decimal to percent:

Multiply the decimal by 100.

$(0.81)(100) = 81\%$

Fraction to percent:

Write a proportion to find an equivalent fraction using 100 as the denominator. The numerator is the percent.

$\frac{4}{5} = \frac{x}{100}$ so $\frac{4}{5} = \frac{80}{100} = 80\%$

Decimal to fraction:

Use the digits in the decimal as the numerator.
Use the decimal place value name as the denominator. Simplify as needed.

a. $0.2 = \frac{2}{10} = \frac{1}{5}$ b. $0.17 = \frac{17}{100}$

Percent to decimal:

Divide the percent by 100.

$43\% \div 100 = 0.43$

Percent to fraction:

Use 100 as the denominator. Use the percent as the numerator. Simplify as needed.

$22\% = \frac{22}{100} = \frac{11}{50}$

$56\% = \frac{56}{100} = \frac{14}{25}$

Fraction to decimal:

Divide the numerator by the denominator.

$\frac{3}{8} = 3 \div 8 = 0.375$ $\frac{5}{8} = 5 \div 8 = 0.625$

$\frac{3}{11} = 3 \div 11 = 0.2727\ldots = 0.\overline{27}$

To see the process for converting repeating decimals to fractions, see problem 2-22 in the *Core Connections, Course 2* text or the Math Notes box referenced above.

Problems

Convert the fraction, decimal, or percent as indicated.

1. Change $\frac{1}{4}$ to a decimal.

2. Change 50% into a fraction in lowest terms.

3. Change 0.75 to a fraction in lowest terms.

4. Change 75% to a decimal.

5. Change 0.38 to a percent.

6. Change $\frac{1}{5}$ to a percent.

7. Change 0.3 to a fraction.

8. Change $\frac{1}{8}$ to a decimal.

9. Change $\frac{1}{3}$ to a decimal.

10. Change 0.08 to a percent.

11. Change 87% to a decimal.

12. Change $\frac{3}{5}$ to a percent.

13. Change 0.4 to a fraction in lowest terms.

14. Change 65% to a fraction in lowest terms.

15. Change $\frac{1}{9}$ to a decimal.

16. Change 125% to a fraction in lowest terms.

17. Change $\frac{8}{5}$ to a decimal.

18. Change 3.25 to a percent.

19. Change $\frac{1}{16}$ to a decimal.
Change the decimal to a percent.

20. Change $\frac{1}{7}$ to a decimal.

21. Change 43% to a fraction.
Change the fraction to a decimal.

22. Change 0.375 to a percent.
Change the percent to a fraction.

23. Change $\frac{7}{8}$ to a decimal.
Change the decimal to a percent.

24. Change $0.\overline{12}$ to a fraction

25. Change $0.\overline{175}$ to a fraction

Answers

1. 0.25
2. $\frac{1}{2}$
3. $\frac{3}{4}$
4. 0.75
5. 38%
6. 20%
7. $\frac{3}{10}$
8. 0.125
9. $0.3\overline{3}$
10. 8%
11. 0.87
12. 60%
13. $\frac{2}{5}$
14. $\frac{13}{20}$
15. $0.1\overline{1}$
16. $\frac{5}{4}$ or $1\frac{1}{4}$
17. 1.6
18. 325%
19. 0.0625; 6.25%
20. $0.\overline{142859}$
21. $\frac{43}{100}$; 0.43
22. $37\frac{1}{2}\%$; $\frac{3}{8}$
23. 0.875; 87.5%
24. $\frac{12}{99} = \frac{4}{33}$
25. $\frac{175}{999}$

ADDITION OF INTEGERS

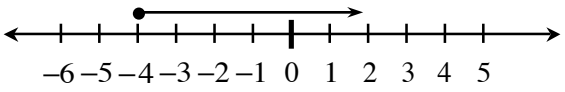
Students review addition of integers using two concrete models: movement along a number line and positive and negative integer tiles.

To add two integers using a number line, start at the first number and then move the appropriate number of spaces to the right or left depending on whether the second number is positive or negative, respectively. Your final location is the sum of the two integers.

To add two integers using integer tiles, a positive number is represented by the appropriate number of (+) tiles and a negative number is represented by the appropriate number of (–) tiles. To add two integers start with a tile representation of the first integer in a diagram and then place into the diagram a tile representative of the second integer. Any equal number of (+) tiles and (–) tiles makes "zero" and can be removed from the diagram. The tiles that remain represent the sum. For additional information, see the Math Notes box in Lesson 2.2.4 of the *Core Connections, Course 2* text.

Example 1

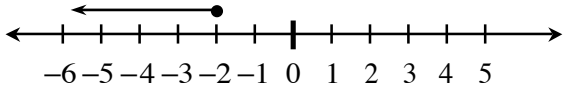
$-4 + 6$



$-4 + 6 = 2$

Example 2

$-2 + (-4)$



$-2 + (-4) = -6$

Example 3

$5 + (-6)$

Start with tiles representing the first number.

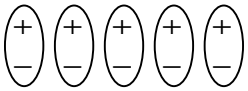
\+ + + + +

Add to the diagram tiles representing the second number.

\+ + + + +

– – – – – –

Circle the zero pairs. –1 is the answer.



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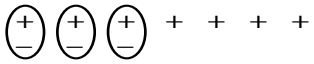
$5 + (-6) = -1$

Example 4

$-3 + 7$



$-3 + 7 = 4$

ADDITION OF INTEGERS IN GENERAL

When you add integers using the tile model, zero pairs are only formed if the two numbers have different signs. After you circle the zero pairs, you count the uncircled tiles to find the sum. If the signs are the same, no zero pairs are formed, and you find the sum of the tiles. Integers can be added without building models by using the rules below.

- If the signs are the same, add the numbers and keep the same sign.
- If the signs are different, ignore the signs (that is, use the absolute value of each number.) Subtract the number closest to zero from the number farthest from zero. The sign of the answer is the same as the number that is farthest from zero, that is, the number with the greater absolute value.

Example

For –4 + 2, –4 is farther from zero on the number line than 2, so subtract: 4 – 2 = 2.
The answer is –2, since the "4," that is, the number farthest from zero, is negative in the original problem.

Problems

Use either model or the rules above to find these sums.

1.	$4+(-2)$	2.	$6+(-1)$	3.	$7+(-7)$
4.	$-10+6$	5.	$-8+2$	6.	$-12+7$
7.	$-5+(-8)$	8.	$-10+(-2)$	9.	$-11+(-16)$
10.	$-8+10$	11.	$-7+15$	12.	$-26+12$
13.	$-3+4+6$	14.	$56+17$	15.	$7+(-10)+(-3)$
16.	$-95+26$	17.	$35+(-6)+8$	18.	$-113+274$
19.	$105+(-65)+20$	20.	$-6+2+(-4)+3+5$	21.	$5+(-3)+(-2)+(-8)$
22.	$-6+(-3)+(-2)+9$	23.	$-6+(-3)+9$	24.	$20+(-70)$
25.	$12+(-7)+(-8)+4+(-3)$	26.	$-26+(-13)$	27.	$-16+(-8)+9$
28.	$12+(-13)+18+(-16)$	29.	$50+(-70)+30$	30.	$19+(-13)+(-5)+20$

Answers

1.	2	2.	5	3.	0	4.	– 4	5.	– 6	6.	–5
7.	–13	8.	–12	9.	–27	10.	2	11.	8	12.	–14
13.	7	14.	73	15.	– 6	16.	– 69	17.	37	18.	161
19.	60	20.	0	21.	–8	22.	–2	23.	0	24.	–50
25.	–2	26.	–39	27.	–15	28.	1	29.	10	30.	20

MULTIPLICATION AND DIVISION OF INTEGERS

Multiply and divide integers two at a time. If the signs are the same, their product will be positive. If the signs are different, their product will be negative.

Follow the same rules for fractions and decimals.

Remember to apply the correct order of operations when you are working with more than one operation.

For additional information, see the Math Notes box in Lesson 3.2.4 of the *Core Connections, Course 2* text.

Examples

a. $2 \cdot 3 = 6$ or $3 \cdot 2 = 6$

b. $-2 \cdot (-3) = 6$ or $(+2) \cdot (+3) = 6$

c. $2 \div 3 = \frac{2}{3}$ or $3 \div 2 = \frac{3}{2}$

d. $(-2) \div (-3) = \frac{2}{3}$ or $(-3) \div (-2) = \frac{3}{2}$

e. $(-2) \cdot 3 = -6$ or $3 \cdot (-2) = -6$

f. $(-2) \div 3 = -\frac{2}{3}$ or $3 \div (-2) = -\frac{3}{2}$

g. $9 \cdot (-7) = -63$ or $-7 \cdot 9 = -63$

h. $-63 \div 9 = -7$ or $9 \div (-63) = -\frac{1}{7}$

Problems

Use the rules above to find each product or quotient.

1.	(–4)(2)	2.	(–3)(4)	3.	(–12)(5)	4.	(–21)(8)
5.	(4)(–9)	6.	(13)(–8)	7.	(45)(–3)	8.	(105)(–7)
9.	(–7)(–6)	10.	(–7)(–9)	11.	(–22)(–8)	12.	(–127)(–4)
13.	(–8)(–4)(2)	14.	(–3)(–3)(–3)	15.	(–5)(–2)(8)(4)	16.	(–5)(–4)(–6)(–3)
17.	(–2)(–5)(4)(8)	18.	(–2)(–5)(–4)(–8)	19.	(–2)(–5)(4)(–8)	20.	2(–5)(4)(–8)
21.	10 ÷ (–5)	22.	18 ÷ (–3)	23.	96 ÷ (–3)	24.	282 ÷ (–6)
25.	–18 ÷ 6	26.	–48 ÷ 4	27.	–121 ÷ 11	28.	–85 ÷ 85
29.	–76 ÷ (–4)	30.	–175 ÷ (–25)	31.	–108 ÷ (–12)	32.	–161 ÷ 23
33.	– 223 ÷ (–223)	34.	354 ÷ (–6)	35.	–1992 ÷ (–24)	36.	–1819 ÷ (–17)
37.	–1624 ÷ 29	38.	1007 ÷ (–53)	39.	994 ÷ (–14)	40.	–2241 ÷ 27

Answers

1.	–8	2.	–12	3.	–60	4.	–168	5.	–36
6.	–104	7.	–135	8.	–735	9.	42	10.	63
11.	176	12.	508	13.	64	14.	–27	15.	320
16.	360	17.	320	18.	320	19.	–320	20.	320
21.	–2	22.	–6	23.	–32	24.	–47	25.	–3
26.	–12	27.	–11	28.	–1	29.	19	30.	7
31.	9	32.	7	33.	1	34.	–59	35.	83
36.	107	37.	–56	38.	–19	39.	–71	40.	–83

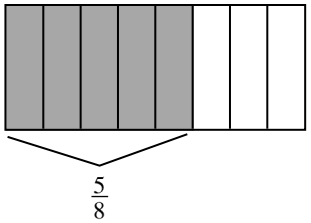
MULTIPLICATION OF FRACTIONS

Multiplication of fractions is reviewed using a rectangular area model. Lines that divide the rectangle to represent one fraction are drawn vertically, and the correct number of parts are shaded. Then lines that divide the rectangle to represent the second fraction are drawn horizontally and part of the shaded region is darkened to represent the product of the two fractions.

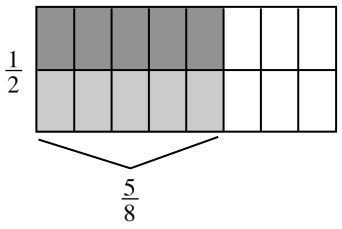
Example 1

$\frac{1}{2} \cdot \frac{5}{8}$ (that is, $\frac{1}{2}$ of $\frac{5}{8}$)

Step 1: Draw a generic rectangle and divide it into 8 pieces vertically. Lightly shade 5 of those pieces. Label it $\frac{5}{8}$.



Step 2: Use a horizontal line and divide the generic rectangle in half. Darkly shade $\frac{1}{2}$ of $\frac{5}{8}$ and label it.



Step 3: Write a number sentence. $\frac{1}{2} \cdot \frac{5}{8} = \frac{5}{16}$

The rule for multiplying fractions derived from the models above is to multiply the numerators, then multiply the denominators. Simplify the product when possible.

For additional information, see the Math Notes box in Lesson 2.2.5 of the *Core Connections, Course 2* text.

Example 2

a. $\frac{2}{3} \cdot \frac{2}{7} \Rightarrow \frac{2 \cdot 2}{3 \cdot 7} \Rightarrow \frac{4}{21}$

b. $\frac{3}{4} \cdot \frac{6}{7} \Rightarrow \frac{3 \cdot 6}{4 \cdot 7} \Rightarrow \frac{18}{28} \Rightarrow \frac{9}{14}$

Problems

Draw an area model for each of the following multiplication problems and write the answer.

1. $\frac{1}{3} \cdot \frac{1}{6}$

2. $\frac{1}{4} \cdot \frac{3}{5}$

3. $\frac{2}{3} \cdot \frac{5}{9}$

Use the rule for multiplying fractions to find the answer for the following problems. Simplify when possible.

4. $\frac{1}{3} \cdot \frac{2}{5} \cdot$

5. $\frac{2}{3} \cdot \frac{2}{7}$

6. $\frac{3}{4} \cdot \frac{1}{5}$

7. $\frac{2}{5} \cdot \frac{2}{3}$

8. $\frac{2}{3} \cdot \frac{1}{4}$

9. $\frac{5}{6} \cdot \frac{2}{3}$

10. $\frac{4}{5} \cdot \frac{3}{4}$

11. $\frac{2}{15} \cdot \frac{1}{2}$

12. $\frac{3}{7} \cdot \frac{1}{2}$

13. $\frac{3}{8} \cdot \frac{4}{5}$

14. $\frac{2}{9} \cdot \frac{3}{5}$

15. $\frac{3}{10} \cdot \frac{5}{7}$

16. $\frac{5}{11} \cdot \frac{6}{7}$

17. $\frac{5}{6} \cdot \frac{3}{10}$

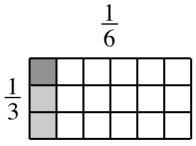
18. $\frac{10}{11} \cdot \frac{3}{5}$

19. $\frac{5}{12} \cdot \frac{3}{5}$

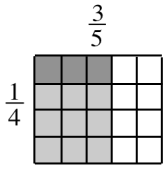
20. $\frac{7}{9} \cdot \frac{5}{14}$

Answers

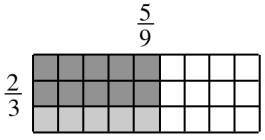
1. $\frac{1}{18}$



2. $\frac{3}{20}$



3. $\frac{10}{27}$



4. $\frac{2}{15}$

5. $\frac{4}{21}$

6. $\frac{3}{20}$

7. $\frac{4}{15}$

8. $\frac{2}{12} = \frac{1}{6}$

9. $\frac{10}{18} = \frac{5}{9}$

10. $\frac{12}{20} = \frac{3}{5}$

11. $\frac{2}{30} = \frac{1}{5}$

12. $\frac{3}{14}$

13. $\frac{12}{40} = \frac{3}{10}$

14. $\frac{6}{45} = \frac{2}{15}$

15. $\frac{15}{70} = \frac{3}{14}$

16. $\frac{30}{77}$

17. $\frac{15}{60} = \frac{1}{4}$

18. $\frac{30}{55} = \frac{6}{11}$

19. $\frac{15}{60} = \frac{1}{4}$

20. $\frac{35}{126} = \frac{5}{18}$